

# Instability in Predictive Matter

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We study the dynamics of a swarm in which agents have full knowledge of the physical laws governing the system they live in. Agents aim to maximize their resource uptake by solving these laws to predict the future of their environment and other agents. The agents also take into account the effects of their and other agents' actions on the environment, which in some cases cause their predictive solutions to not converge. We conclude that predictivity pays off when the degree of predictivity is small, or when the number of predictors in the population are small compared to the non-predictors.

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One of the steadfast pillars of science is the notion of causality. The past determines the future. However, the fact that the world can be *correctly* modeled and predicted leads to an interesting complication: an intelligent system can understand the laws governing its world, infer the outcomes of available choices, and decide on how to behave in the present according to the future. As such, an intelligent system can be subject to a more stringent kind of causality where not only does its behavior in the past determines the future, but also a predicted future determines its behavior in the present. Interestingly, in some cases the very act of predicting or describing the world can negate the prediction or description, imposing a fundamental limit on the domain of applicability of the scientific method [1].

Many-body physics has extended its applicability from equilibrium systems [2] to externally driven systems [3], to self-driven systems [4, 5]. Can we extend this domain further, from describing passive, reactive and active states of matter, towards “intelligent” states of matter? What laws and constraints govern these states?

Predictivity permeates social and biological systems. One example is a walking crowd. It has been found that walking people chose paths that will not result in a collision within the next few seconds [6]. A second familiar predictive system is the stock market. Since price changes are driven by the amount of surprise created by financial news, in order to beat the returns of others, investors must predict both the news and the reaction of others to the news [7, 8]. A third example is a prey evading a group of predators. As in the former two examples the intelligent agents here must base their motion not only on a predicted future, but also on the predicted future of other fellow predictors.

The field of differential game theory aids in describing systems in which strategy and anticipation plays an important role [9, 10]. Differential games, however, typically concern with a finite number of discrete actors (typically two) instead of a continuous distribution of actors. Furthermore these games are typically formulated solely

in terms of the interaction between agents rather than taking into account realistic physical forces and environments.

Our purpose here is to investigate the dynamics of a predictive population competitively drawn to a resource, and determine under what conditions and to what extent their predictive capability breaks down.

**The Model.** We study the statistical mechanical properties of a predictive population  $n(\vec{r}, t)$  every member of which aiming to maximize the uptake of a spatio-temporal resource  $\phi(\vec{r}, t)$ . For the sake of concreteness, we assume that  $\phi$  diffuses, and is absorbed by the agents.

$$\frac{d\phi}{dt} = D\nabla^2\phi - \gamma\phi \cdot n(\vec{r}, t) \quad (1)$$

We are interested a model in which all agents have full knowledge of this equation, and can solve it exactly “in their head”. To this end, we assume that the velocity  $\vec{v}(\vec{r}, t) = c\hat{u}$ , of a predictive agent at position  $\vec{r}$  and time  $t$ , has constant magnitude  $c$ , and variable direction,  $\hat{u} = \vec{M}/|\vec{M}|$ , determined by the future resource as

$$\vec{M}(\vec{r}, t) = \int_t^{t+\tau} \int_{|\vec{r}' - \vec{r}| < v \cdot \tau} d^3\vec{r}' \cdot \phi(\vec{r}', t') \hat{w}(\vec{r}') F(\vec{r} - \vec{r}'). \quad (2)$$

where  $R = \sqrt{(\vec{r}' - \vec{r})^2 + \lambda(t' - t)^2}$ ,  $\lambda$  is a constant with units  $[L]/[T]$ ,  $d^3R = d^2r' dt'$ ,  $\hat{w}(\vec{r}') = (\vec{r}' - \vec{r})/|\vec{r}' - \vec{r}|$  is a unit vector,  $F(R)$  is a force scaling, generally taken to be  $1/R^2$ , and  $\tau$  and  $\sigma$  are the temporal and spatial sight radii respectively. In other words, the predictive agents are drawn to areas of high resource concentration, both in the present and in the future, only considering the resource in regions of space that they can travel to in the given amount of time.

We emphasize that we do not explicitly define the internal physical mechanisms allowing the agents to make predictions, which, unlike our equations, must be strictly causal. Instead we assume that these mechanisms, whatever they are, will yield *correct* predictions, allowing the

agents to couple to the literal future. Accordingly, we seek a solution for every individual trajectory that is self-consistent with (1) and (2). Such self-consistent solutions will not always exist, since causality breaks down when the behavior of an agent leads to a failure of its own prediction. From the point of view of the agents, such systems are intrinsically unpredictable, and thus, practically speaking, metaphysical. One of our tasks is to determine when and to what degree causality contradicts with predictability, and how this influences the success of agents.

We solve equations (1) and (2) iteratively. First, the agents consider what would happen if the population simply ascends gradients of the resource field (i.e.  $\tau = 0$ ). This yields a zeroth order estimate for the trajectory  $v^{(0)}(x, t)$  of the agents, which in turn determines the response of the resource trajectory  $\phi^{(0)}(x, t)$  according to (2). This gives a finer, first order estimate of everyone's trajectory  $v^{(1)}(x, t)$ , which in turn determines a finer estimate for the response of the resource trajectory  $\phi^{(1)}(x, t)$ . The agents repeat this procedure for higher order estimations of  $v$  and  $\phi$  for a large fixed number of steps. Throughout, we refer to the index  $i$  of  $\phi^{(i)}$  and  $v^{(i)}$  as the "iteration" coordinate.  $i$  can be thought as an internal degree of freedom of agents.

If the solutions  $\phi^{(i)}$  and  $v^{(i)}$  converge with iteration  $i$ , (or fluctuate with small amplitude), this indicates that the agent can exactly (or approximately) predict the future. If the iteration diverges (or fluctuates with a large amplitude), the agent cannot make a reliable, self consistent prediction.

The last point of the iteration is taken to be the *actual* realization of events, i.e. after a certain large number of iterations, the agents act with or without a mature prediction. In this sense, the motion of all agents are already determined as soon as the agents finish computing and start moving.

Note that there is no guarantee that even a accurate self-consistent prediction of the future will help an agent consume larger amounts of the resource: a self-consistent solution may be less successful, in terms of total resource consumption, than either a non-self-consistent solution or a non-predictive ( $\tau = 0$ ) solution.

We perform a large number of simulations to determine if and how well predictive solutions converge, and how predictive agents perform compared to non-predictive ( $\tau = 0$ ) agents. Except when otherwise stated, the consumption rate of the agents (cf. Eqn.1) is  $\gamma = 1$ , the diffusion rate is  $D = 0.01$ , the velocity of the agents is  $v = 1$ , and the time step is  $\epsilon = 0.001$ .

When agents are unable to converge to a decision, they tend to cycle between few possible trajectory options. To quantify the degree of prediction convergence we take the discrete Fourier transform of an agent's calculated paths as a function of  $i$  and look for peaks corresponding to agents' indecision between several possible choices. To

quantify the "success" of an agent we record its total resource uptake.

For an analysis of how an agent's temporal sight effects performance, we run the simulation with two initial conditions for the resource distribution: one set of simulations start with two resource peaks of different magnitude, and the other set starts with a resource distributed with random (Perlin) noise. In both cases  $N = 5000$  agents are uniformly distributed across a two dimensional domain with periodic boundary conditions. We use a small consumption rate to ensure that the agents will do not exhaust all available resources during the simulation time, allowing us to see changes in the consumption factor as a function of degree of predictivity, and as a function of the fraction of predictive agents in a mixed population of predictive and non-predictive (gradient ascending) agents.

Instead of reporting the absolute amount of resources consumed, we obtain a "consumption factor" for the predictive and gradient agents based on the amount of resources consumed per predictive and gradient agent. We normalize the amount of resources consumed by each class of agent the number of that type of agents and by the amount of resources an agent would consume on average if the agents had consumed all the resource. To be specific, the consumption factor is

$$C_{p,g} = \frac{R_{p,g}/N_{p,g}}{R_{tot}/(N_p + N_g)}$$

where  $C_{p,g}$  is the consumption factor of the predictive or gradient agents (as indicated by the subscript),  $R_p$  and  $R_g$  are the amounts of resources consumed,  $N_p$  and  $N_g$  are the numbers of predictive and gradient agents, and  $R_{tot}$  is the total amount of resources available for consumption. The consumption factor allows us to easily compare how well the agents do relative to each other, and simply reduces to the fraction of the total resources consumed when one of  $N_p$  or  $N_g$  is zero.

### Results: Predictive Population.

We determine how the amount of resource consumption changes as a function of solution iteration. For a population consisting purely of predictive agents we show the amount of consumption for a selection of individual agents' consumption factors in Fig. 1.

We find, counterintuitively, that large predictivity is detrimental. In general, we observe that as the predictivity increases, the average consumption factor decreases and the fluctuation amplitude with the iteration coordinate increases. Populations that aim to see further into the future are more indecisive, as quantified by their solution convergence; and less successful, as quantified by their total resource uptake.

To detect periodicity in the iteration coordinate  $i$ , we take the Fourier transform of the  $x$  component of an agents' position at a random time in the middle of its trajectory and find what the largest Fourier component

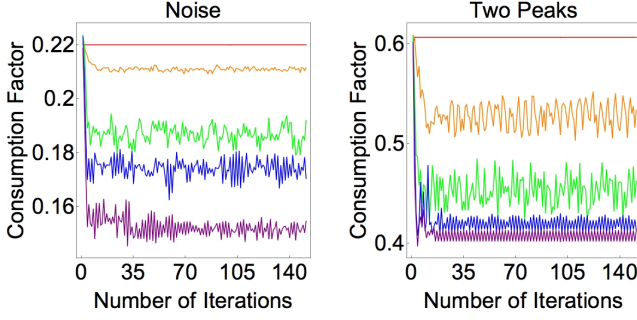


FIG. 1. (Color Online) Consumption factor vs. solution iteration for predictive agents on a random (Perlin noise) distribution. There are 5000 agents had predictivities 0 (red), 0.02 (orange), 0.05 (green), 0.09 (blue), 0.15 (purple). The left figure is for a random resource, the right figure is for the two peaks resource.

is. The point plots in Fig. 2 shows the initial condition dependence of the period and amplitude of these Fourier components. Notice that agents with similar Fourier components and oscillation cluster together, forming spatial domains. It is reasonable that agents that are close together compute similar trajectories and have similar degrees of indecisiveness.

**Results: Mixed Population.** An additional question we address is how well predictive agents perform when competing against non-predictive agents when the two are mixed together. Non-predictive agents are equivalent to predictive agents with  $\tau = 0$ , and move according to a gradient ascent rule; i.e. they look at their surrounding sites and move in the direction of greatest increase of resource.

We run simulations where the total number of agents are kept constant while the fraction of predictive agents is varied. Again, we report the consumption factor for the predictive and gradient agents.

Studying Fig. 3A, we observe that in the limit where there are very few predictive agents, the non-predictors are outperformed by the predictors. However, as the number of predictive agents increases, their advantage is mitigated, and they start performing more poorly than the non-predictors. It is interesting to note that while in the limit of very few predictors, increasing the predictivity increases the consumption factor of the agents (e.g. the magenta line in Fig. 3B), there is a region where while increasing the predictivity of the agents is no longer beneficial (e.g. the other lines in Fig. 3B), it is still better to be a predictive agent than a gradient agent. Eventually the lines do cross and the gradient agents gain the advantage.

Thus, predictive agents aiming to see further into the future perform better against the non-predictors only when the non-predictors are in the majority. Our explanation is, when there are few predictive agents, their

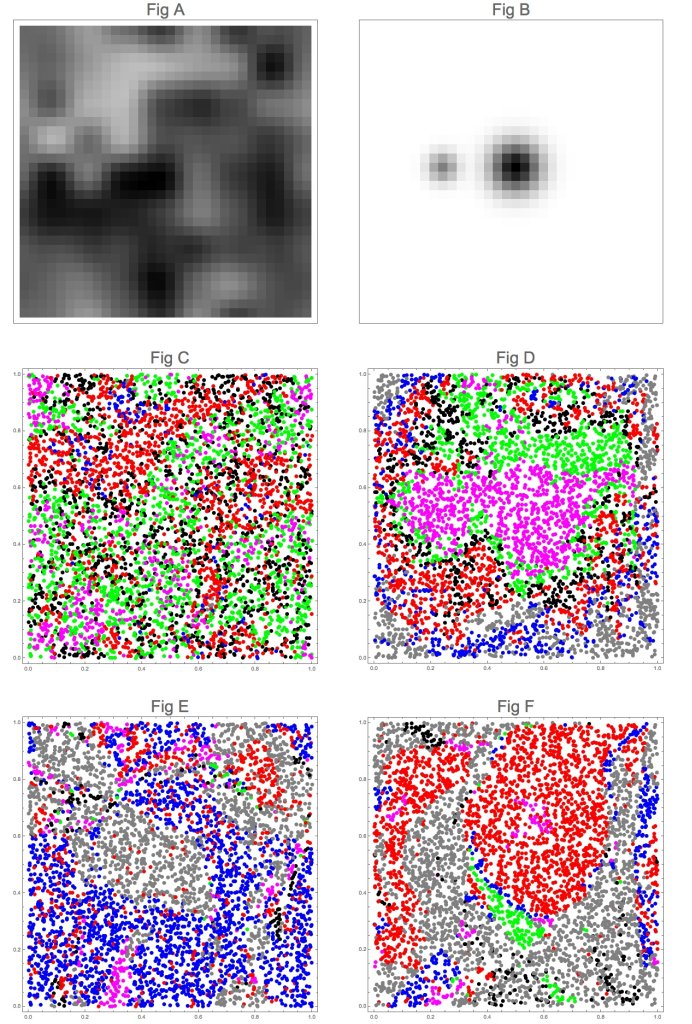


FIG. 2. (Color Online) We initialized the system with 5000 randomly scattered predictors with  $\tau = 0.1$  and analyzed the degree of predictive self consistency as a function of initial condition. We explored two initial conditions for the resource distribution (shown in panels A, B): Perlin Noise (Left column) and two unequal peaks (Right column). Panels C, D show the oscillation amplitude of individual agent's consumption factors with the oscillation small to large of magenta, green, black, red, blue, gray. Panels E, F show the period corresponding to the largest fourier component. For panel E, 1-3 (magenta), 4-6 (green), 7-9 (black), 10-12 (red), 13 (blue), and 14-15 (gray). For panel F, 1-4 (magenta), 5-8 (green), 9-11 (black), 13-16 (red), 17-18 (blue), and 19-20 (gray).

actions do not significantly affect the future no matter what they do, so they are able to predict the future self-consistently (and thus reliably) to pick the best option for their motion.

Investigating the simulation videos of many-body trajectories corresponding to consecutive computation iterations reveal the mechanism behind the prediction driven instability. To maximize resource uptake, agents must move along trajectories that are not only high in re-

sources, but low in population. Thus, it can be advantageous to move towards lower, “next best” peaks if these should end up drawing lesser number of agents. When the density of predictors is low, they can easily see ahead which lesser peaks will be unpopular, directly move there, and out-compete every other agent who moves to the same highest peak.

However when the predictors are densely packed, all predictors will pick the same “next-best” peaks which they anticipate will be least popular. Since this anticipation now increases the peak’s future popularity, in the next iteration the predictors will target yet another peak, or return to an earlier choice.

In the end, predictors in close vicinity cycle between the same options, and they all end up in (the last iteration of) what they think is will be unpopular. Near-by predictors destabilize the solutions of one other. The larger  $\tau$ , the more aware agents become of further competitors for the same peak and more likely will they consider lesser and further peaks. In the mean time, non-predictive agents go directly towards the nearest, highest peak, and consume it together.

Our model could be viewed in the light of game theory: every possible trajectory  $C_i$  is a strategy, and  $\int_{C_i} \gamma \phi(\vec{r}, t) dr$  is the corresponding payoff. When players pick close-by trajectories they get lesser payoff. As a result we have a multi-strategy / multi-player generalization of an anti-coordination game, such as snowdrift or hawk-dove, in which there is no unique Nash equilibrium.

The scale of parameters  $\tau$  and  $\sigma$ , together with the initial density determine the number of players that participate in the trajectory game.

**Conclusion.** We have constructed a simple model in which agents attempt to predict the future and act on the prediction to determine their present course of action. We find that convergence of solutions become unlikely with increased predictivity. Additionally we find that the periodicity (as a function of initial position) can cluster into distinct domains (cf. Fig. 2). Despite predictive agents’ inability to find convergent solutions when in a homogeneous predictive population, when small amounts of predictive agents compete against gradient agents, they outperform the gradient agents (cf. Fig. 3).

We expect this instability to be a general property for all systems that strongly couple to more than one predicting constituent. When the constituents of a system base their actions on their prediction of the future, which itself is affected by those actions, the system should exhibit high sensitivity to initial conditions and temporal sight of agents.

Whenever a predicting agent couples to its subject, we expect that this (1) compromises the degree of predictability of the system (2) causes an instability in the asymptotic behavior of the system. The larger the number of predictors, the stronger they couple to the system;

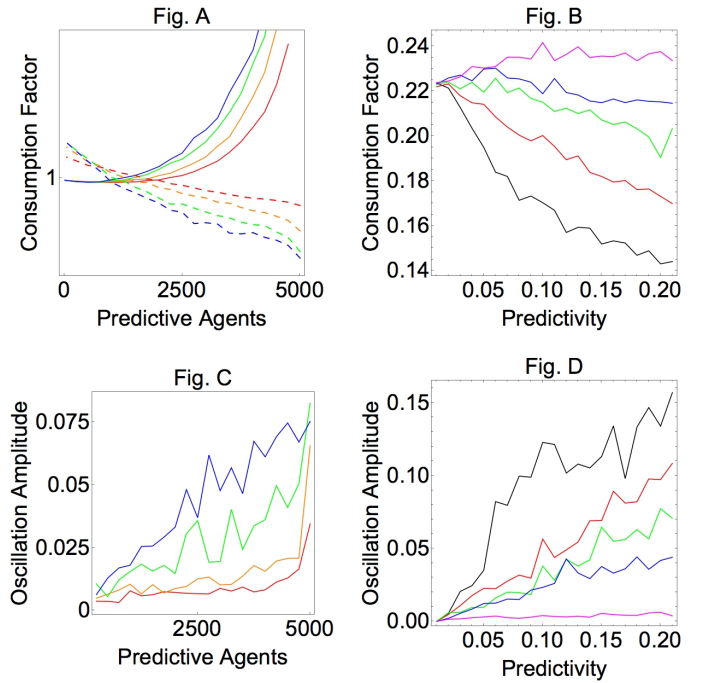


FIG. 3. (Color Online) All tests done on a noise resource with 5000 total agents, some predictive and some gradient. *Panel A, C:* In panel A, dashed lines represent the consumption of predictive agents, solid lines represent gradient agents. There are 5000 total agents, different lines represent populations of agents with different predictivities:  $\tau = 0.03$  (red),  $\tau = 0.05$  (orange),  $\tau = 0.08$  (green),  $\tau = 0.1$  (blue). *Panel B, D:* The lines represent different total fractions of predictive agents: 100% predictive agents (black), 80% predictive agents (red), 50% predictive agents (green), 20% predictive agents (blue), and 1% predictive agents (magenta). A consumption factor of 0.05 was used here.

the further into the future they aim to predict, the less predictable and unstable the system will become.

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